Engineering Analysis
Final Exam

June 7, 2018

1. (15 pts) Determine the Gaussian quadrature formula for the integral

$$\int_{-1}^{1} f(x)dx$$

when three Gaussian nodes and three weights are used.

2. (15 pts) Let F be a function of two variables x and y whose gradient at $(0,0)$ is $[-5,1]^T$ and whose Hessian is

$$\begin{bmatrix} 6 & -1 \\ -1 & 2 \end{bmatrix}.$$

Estimate the minimum point of F. Explain your solution procedure.

3. (15 pts) Maximize

$$z = 2x_1 + 3x_2 + x_3$$

subject to

$$\begin{aligned} x_1 - 2x_2 + x_3 &\leq 6, \\ 2x_1 + 4x_2 + x_3 &\leq 12, \\ 3x_1 + 3x_2 - x_3 &\leq 5, \end{aligned}$$

Use the simplex method.

4. (20 pts) Suppose that we have a set of five measured data for I and v as follows:

$$(I, v) = (10.27, 15.23), (20.12, 20.07), (30.20, 22.54), (40.17, 24.06), (50.24, 25.04).$$

In order to approximate the data, we want to use the following expression

$$v = \frac{\alpha}{1 + \beta/I}.$$

Then, find the coefficients α and β that make the expression fit the observed data in the least squares sense.

5. (20 pts) Consider a heat transfer problem through a thin plate. The 1D steady state solution u to the problem is the solution to the following differential equation

$$\frac{d^2u}{dx^2} = q, \quad 0 \leq x \leq \omega,$$

where ω is the thickness and q is a constant. The boundary conditions are

$$u(x = 0, t) = u_0, \quad u(x = \omega, t) = 0.$$

Assume that $q = 1$, $u_0 = 1$ and $\omega = 1$. Use five discrete points $i = 0, 1, \cdots, 4$ so that $h = \Delta x = \omega/4 = 0.25$. Using the finite difference method, find the approximate solution to the differential equation at the discrete points.

6. (15 pts) Consider a two-point boundary value problem:

$$x'' = f(t, x, x'), \quad x(a) = \alpha, \quad x(b) = \beta.$$

The procedure of the shooting method starts with solving the initial-value problem

$$x'' = f(t, x, x'), \quad x(a) = \alpha, \quad x'(a) = z$$

from $t = a$ to $t = b$. Let the value of the solution at b be denoted by $\psi(z)$. Then, the objective is to adjust z until $\psi(z) = \beta$. Determine the function $\psi(z)$ explicitly in the case of the following two-point boundary value problem

$$x'' = x, \quad x(-1) = e, \quad x(1) = \frac{1}{2}e.$$