Numerical Analysis and Visualization

Homework 6

May 17, 2017

1. For the function $f(t, y) = t^2 y + 1$, answer the following questions. Does f satisfy a Lipschitz condition on $D = \{(t, y) | 0 \leq t \leq 1, -\infty < y < \infty\}$?

2. Given the initial value problem

$$y' = -y + t + 1, \quad 0 \leq t \leq 5, \;\; y(0) = 1,$$

with exact solution $y(t) = e^{-t} + t$.

   (a) Then approximate $y(5)$ using Euler's method with $h = 0.2$, $h = 0.1$, and $h = 0.05$. Compare the approximate results with the exact one and analyze them.

   (b) Use Taylor's method of order two with $h = 0.1$ to approximate the solution $y(5)$, and compare it with the actual value.

3. The boundary-value problem

$$y'' = 4(y - x), \quad 0 \leq x \leq 1, \;\; y(0) = 0, \;\; y(1) = 2,$$

has the solution $y(x) = e^2(e^4-1)^{-1}(e^{2x}-e^{-2x})+x$. Use the Linear Shooting method to approximate the solution, and compare the results to the actual solution with $h = 1/2$.

4. Use the Finite Difference method to approximate the solution to

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4, \;\; 0 < x < 1, \;\; 0 < y < 2;$$

$$u(x, 0) = x^2, u(x, 2) = (x - 2)^2, 0 \leq x \leq 1;$$
$$u(0, y) = y^2, u(1, y) = (y - 1)^2, 0 \leq y \leq 2.$$

   Use $h = 1/2$, and compare the results to the actual solution $u(x, y) = (x - y)^2$.